PHY2202S PHYSICS OF ATOMS & MOLECULES Final Exam 2012 Take-home: 19 April 2011, 12 noon – 20 April 2012, 5pm R.S. Marjoribanks

Restrictions: You may not cooperate with or take help from others in answering these questions. You may consult any 'passive' reference (books, online references), and use computer-based aids. Please note in your answers any aid you take from Wolfram Alpha. General: Be precise and careful in your written answers, identifying assumptions and giving all logical steps. Reference to any sources you use.

ANSWER ALL QUESTIONS

[10] 1. Neglecting relativistic and other small effects, the energies of the s, p, d... states of hydrogen for a given value of the principle quantum number n are the same. They have the same potential energy, and the same kinetic energy. However, say a student argues as follows: "Compared with high-angular-momentum states of the same n, the low-angular-momentum states have a higher probability of being near the nucleus, so their potential energy must be more negative. Their low angular momentum means they have lower kinetic energy also. So the low-angular-momentum states should lie deeper in energy."

Give an explanation for the student about what is wrong with this argument.

- [5] 2. Which of the following levels do not exist in hydrogen, and why? 3f ${}^{2}F_{3/2}$, 3p ${}^{2}P_{3/2}$, 3s ${}^{2}S_{3/2}$, 3d ${}^{3}D_{2}$, 2p ${}^{2}D_{3/2}$,
- [20] 3. In the single-electron Hamiltonian, atomic spin-orbit coupling can be represented by a term:

$$\frac{1}{2m^2c^2}\frac{1}{r}\frac{\partial V(r)}{\partial r}\vec{l}\cdot\vec{s}$$

where r is the radial coordinate of the electron, m is the electron mass, l the orbital angular momentum operator and s the electron spin operator. V(r) is the central electrostatic potential energy.

- a) Give simple arguments which justify the form of this expression.
- b) If V(r) is assumed to be the Coulomb interaction with a point nucleus of atomic number Z, then for a 2p electron

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{24} \left(\frac{Z}{a_0} \right)^3$$
, where $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2}$ is the Bohr radius

Show that under the above assumptions, the 2p spin-orbit splitting is given by

$$E\left(j=\frac{3}{2}\right) - E\left(j=\frac{1}{2}\right) = \frac{1}{32}(Z\alpha)^4 mc^2$$

where α is the fine-structure constant.

When negative muons are captured in atoms, characteristic X-rays are emitted. In muonic ${}_{26}^{56}Fe$ the $2p_{3/2} \rightarrow 1s_{1/2}$ transition energy is 1257.0 keV and for $2p_{1/2} \rightarrow 1s_{1/2}$ 1252.9 keV. In muonic ${}_{82}^{206}Pb$ the corresponding energies are 5974.0 and 5788.3 keV.

c) Compare these values with those which would obtain for a point nucleus. Comment on any discrepancies.

[20] 4. We discussed a term of the form $A(J)\hat{\vec{I}}\cdot\hat{\vec{J}}$ included in the atomic Hamiltonian in order to account for part of the interaction between the nucleus and the atomic electrons. What does the term describe, and why does it have this form?

Hyperfine levels with quantum numbers F and F-1 are separated in energy by $\Delta E_{F,F-1}$. Show that the hyperfine levels associated with a given electronic level should obey the *Interval Rule*:

$$\frac{\Delta E_{F,F-1}}{\Delta E_{F-1,F-2}} = \frac{F}{F-1}$$

It commonly happens that the hyperfine structure in a transition is dominated by the contribution of an s-electron in one of the levels. What property of s-electrons could be responsible for this?

A transition of this kind occurs at 494 nm in singly ionized ¹³³Cs between a level from the $5p^56s$ configuration and one from the $5p^56p$ configuration. Five hyperfine structure components are observed with wavenumbers relative to that with the lowest wavenumber as follows: 0.0, 8.1, 19.5, 33.7, 51.3 m⁻¹. The experimental uncertainty in the position of each component is of order 0.1 m⁻¹. Find the nuclear spin of ¹³³Cs, and the value of *J* for the level arising from the $5p^56s$ configuration.

[25] 5. Alkali spectroscopy

a) Explain what is meant by the term *quantum defect*, used in the description of alkali energy levels. Give a brief qualitative account of how you would expect the quantum defect to vary.

- i) from lower to higher values of the orbital quantum number l in a given alkali
- ii) from lighter to heavier alkalis for a given value of *l*.

The sodium atom has atomic number Z = 11. Write down the complete configuration of the ground (3s) level. Find the quantum defect of this level, given that the first ionization potential of sodium is 5.14 eV.

For the next part we're concerned only with gross structure, and neglect spin-orbit interaction — this effect splits most of the energy levels of sodium into closely-spaced doublets, but we treat each doublet as a single level here.

b) Sodium has a single series of absorption lines. Make use of the data in (a) to find the wavelength of the series limit. The longest-wavelength absorption line (3s - 3p) is at $\lambda_1 = 589$ nm. Find the quantum defect of the 3p level. Hence estimate the wavelength λ_2 of the second line in the series (3s - 4p).

Sodium atoms are illuminated with light of wavelength around λ_2 , so that they are excited to the 4p level. Make a rough copy of the enough of the energy-level diagram to indicate the routes by which the atoms may return to the ground level. You should find that the fluorescence spectrum contains 6 spectral lines, two of which are the transitions at λ_1 and λ_2 . Two of the other have wavelengths of 1.14 μ m and 2.21 μ m. Again using the properties of the quantum defect, identify these latter two transitions on your energy-level diagram.

Use the information given to improve your estimate of λ_2 .

[20] 6. Zeeman Effect

a) What do the quantum numbers L, S, J, M_J signify in the LS coupling model of multielectron atoms? State and give a simple justification of the electric dipole selection rules in atomic spectroscopy which apply in the LS coupling scheme.

Suggest a simple physical reason why a transition between two levels both having $M_J = 0$ is forbidden for the case $\Delta I = 0$.

b) Explain briefly the role of the Landé g-factor, given by

$$g_J = \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

in describing the weak-field Zeeman effect (no formal derivation required).

c) The weak-field Zeeman splitting pattern of a transition from an upper level *known* to be ³D to a lower level of *unknown L*, *S*, *J* values is shown in the line spectrum below. The spectrum shows all possible components of the transition. Find the *J* value of the upper level and the *L*, *S*, *J* values of the lower level.



ν

[100] TOTAL MARKS